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ASYMPTOTICS OF THE GAUSS HYPERGEOMETRIC FUNCTION WITH LARGE PARAMETERS, I

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Abstract. We obtain asymptotic expansions for the Gauss hypergeometric function

$$F(a+\varepsilon_1\lambda, b+\varepsilon_2\lambda; c+\varepsilon_3\lambda; z)$$

as $|\lambda|\to\infty$ when the ε_j are finite by an application of the method of steepest descents, thereby extending previous results corresponding to $\varepsilon_j = 0, \pm 1$. By means of connection formulas satisfied by F it is possible to arrange the above hypergeometric function into three basic groups. In Part I we consider the cases (i) $\varepsilon_1 > 0$, $\varepsilon_2 = 0$, $\varepsilon_3 = 1$ and (ii) $\varepsilon_1 > 0$, $\varepsilon_2 = -1$, $\varepsilon_3 = 0$; the third case ε_1, $\varepsilon_2 > 0$, $\varepsilon_3 = 1$ is deferred to Part II. The resulting expansions are of Poincaré type and hold in restricted domains of the complex z-plane. Numerical results illustrating the accuracy of the different expansions are given.

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