

ASYMPTOTICS OF THE GAUSS HYPERGEOMETRIC FUNCTION WITH LARGE PARAMETERS, II

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Abstract. We obtain asymptotic expansions by application of the method of steepest descents for the Gauss hypergeometric function

$$F(a + \varepsilon_1 \lambda, b + \varepsilon_2 \lambda; c + \lambda; z)$$

as $|\lambda| \rightarrow \infty$ when $0 < \varepsilon_1 < 1$ and $\varepsilon_2 > 1$ where, without loss of generality, it is supposed that $\varepsilon_1 \leq \varepsilon_2$. The resulting expansions are of Poincaré type and break down in the neighbourhood of certain critical points in the z -plane. Numerical results illustrating the accuracy of the different expansions are given.

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