EXTENSION OF SOME CLASSICAL SUMMATION THEOREMS FOR THE GENERALIZED HYPERGEOMETRIC SERIES WITH INTEGRAL PARAMETER DIFFERENCES

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Abstract. We derive extensions of the classical summation theorems of Kummer and Watson for the generalized hypergeometric series where r pairs of numeratorial and denominatorial parameters differ by positive integers. The results are obtained with the help of a generalization of Kummer's second summation theorem for the $_2F_1$ series given recently by Rakha and Rathie [Integral Transforms and Special Functions, **22**, 823–840 (2011)] together with generalizations of the Euler transformations for the $_{r+2}F_{r+1}(z)$ function. A few interesting special cases are also presented.

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REFERENCES

[1] J. CHOI, A. K. RATHIE AND H. M. SRIVASTAVA, *A generalization of a formula due to Kummer*, Integral Transforms and Special Functions **22**, 851–859, 2011.

[2] C. FOX, *The expression of hypergeometric series in terms of similar series*, Proc. London Math. Soc. **26**, 201–210, 1927.

[3] P. W. KARLSSON, *Hypergeometric functions with integral parameter differences*, J. Math. Phys. **12**, 270–271, 1971.

[4] Y. S. KIM, M. A. RAKHA AND A. K. RATHIE, *Extensions of certain classical summation theorems for the series $_2F_1$, $_3F_2$ and $_4F_3$ with applications in Ramanujan's summations*, Int. J. Math. & Math. Sci., Article ID 309503, 26 pages, 2010.

[5] Y. S. KIM, A. K. RATHIE, U. PANDEY AND R. B. PARIS, *Some summation formulas for the hypergeometric series $_{r+2}F_{r+1}(\frac{1}{2})$*, Hacettepe J. Math. Stat. **42**, 281–287, 2013.

[6] Y. S. KIM, A. K. RATHIE AND R. B. PARIS, *An extension of Saalschütz's summation theorem for the series $_{r+3}F_{r+2}$*, Integral Transforms and Special Functions **24** (11), 916–921, 2013.

[7] E. E. KUMMER, *Über die hypergeometrische Reihe*

$$1 + \frac{\alpha \cdot \beta}{1 \cdot \gamma} x + \frac{\alpha(\alpha+1) \cdot \beta(\beta+1)}{1 \cdot 2 \cdot \gamma(\gamma+1)} x^2 + \frac{\alpha(\alpha+1)(\alpha+2) \cdot \beta(\beta+1)(\beta+2)}{1 \cdot 2 \cdot 3 \cdot \gamma(\gamma+1)(\gamma+2)} x^3 + \cdots,$$

J. Reine Angew. Math. **15**, 39–93 and 127–172, 1836.

[8] J. L. LAVOIE, F. GRONDIN AND A. K. RATHIE, *Generalizations of Watson's theorem on the sum of a $_3F_2$*, Indian J. Math. **34**, 23–32, 1992.

[9] J. L. LAVOIE, F. GRONDIN AND A. K. RATHIE, *Generalizations of Dixon's theorem on the sum of a $_3F_2$*, Math. Comp. **62**, 267–276, 1994.

[10] J. L. LAVOIE, F. GRONDIN AND A. K. RATHIE, *Generalizations of Whipple's theorem on the sum of a $_3F_2$*, J. Comput. Appl. Math. **72**, 293–300, 1996.

[11] Y. L. LUKE, *Mathematical Functions and Their Approximations*, Academic Press, New York, 1975.

[12] R. S. MAIER, *P-symbols, Heun identities, and $_3F_2$ identities*, Contemp. Math. **471**, 139–159, 2008.

[13] A. R. MILLER, *Certain summation and transformation formulas for generalized hypergeometric series*, J. Comp. Appl. Math. **231**, 964–972, 2009.

[14] A. R. MILLER AND R. B. PARIS, *Certain transformations and summations for the generalized hypergeometric series with integral parameter differences*, Integral Transforms and Special Functions, **22**, 67–77, 2011.

[15] A. R. MILLER AND R. B. PARIS, *Euler-type transformations for the generalized hypergeometric function $_{r+2}F_{r+1}(x)$*, Zeitschrift Angew. Math. Phys. **62**, 31–45, 2011.

[16] A. R. MILLER AND R. B. PARIS, *On a result related to transformations and summations of generalized hypergeometric series*, Math. Communications **17**, 205–210, 2012.

[17] A. R. MILLER AND R. B. PARIS, *Transformation formulas for the generalized hypergeometric function with integral parameter differences*, Rocky Mountain J. Math. **43**, 291–327, 2013.

[18] A. R. MILLER AND H. M. SRIVASTAVA, *Karlsson–Minton summation theorems for the generalized hypergeometric series of unit argument*, Integral Transforms and Special Functions **21**, 603–612, 2010.

[19] M. A. RAKHA AND A. K. RATHIE, *Generalizations of classical summation theorems for the series $_2F_1$ and $_3F_2$ with applications*, Integral Transforms and Special Functions **22**, 823–840, 2011.

[20] A. K. RATHIE AND R. B. PARIS, *An extension of the Euler-type transformation for the $_3F_2$ series*, Far East. J. Math. Sci. **27**, 43–48, 2007.

[21] L. J. SLATER, *Generalized Hypergeometric Functions*, Cambridge University Press, Cambridge, 1966.

[22] H. M. SRIVASTAVA, *Generalized hypergeometric functions with integral parameter differences*, Nederl. Akad. Wetensch. Indag. Math. **35**, 38–40, 1973.

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