Journal of Classical Analysis
Volume 4, Number 1 (2014), 1–39
doi:10.7153/jca-04-01

THE RESURGENCE PROPERTIES OF THE LARGE ORDER ASYMPTOTICS OF THE ANGER—WEBER FUNCTION I

GERGŐ NEMES

Abstract. The aim of this paper is to derive new representations for the Anger–Weber function, exploiting the reformulation of the method of steepest descents by C. J. Howls (Howls, Proc. R. Soc. Lond. A **439** (1992) 373–396). Using these representations, we obtain a number of properties of the large order asymptotic expansions of the Anger–Weber function, including explicit and realistic error bounds, asymptotics for the late coefficients, exponentially improved asymptotic expansions, and the smooth transition of the Stokes discontinuities.

Mathematics subject classification (2010): 41A60, 30E15, 34M40.
Keywords and phrases: asymptotic expansions, Anger–Weber function, error bounds, Stokes phenomenon, late coefficients.

REFERENCES

[1] M. V. BERRY, *Uniform asymptotic smoothing of Stokes' discontinuities*, Proc. Roy. Soc. London Ser. A **422** (1989), 7–21.
[2] R. B. DINGLE, *Asymptotic Expansions: Their Derivation and Interpretation*, Academic Press, London/New York, 1973.
[3] C. J. HOWLS, *Hyperasymptotics for integrals with finite endpoints*, Proc. Roy. Soc. London Ser. A **439** (1992), 373–396.
[4] H. A. LAUWERIER, *The calculation of the coefficients of certain asymptotic series by means of linear recurrent relations*, Appl. Sci. Res. Sec. B **2** (1952), 77–84.
[5] C. S. MEIJER, *Über die asymptotische Entwicklung von* $\int_0^{\infty - i(\arg w - \mu)} e^{\nu z - w \sinh z} dz$ $\left(-\frac{\pi}{2} < \mu < \frac{\pi}{2}\right)$ *für grosse Werte von* $|w|$ *und* $|\nu|$, Proc. Kon. Akad. Wet. Amsterdam **35** (1932), 1170–1180 and 1291–1303.
[6] L. M. MILNE-THOMSON, *The Calculus of Finite Differences*, Macmillan and Co. Ltd., London, 1933.
[7] G. NEMES, *The resurgence properties of the large-order asymptotics of the Hankel and Bessel functions*, Anal. Appl. **12** (2014), 403–462.
[8] N. E. NÖRLUND, *Vorlesungen über Differenzenrechnung*, Springer-Verlag, Berlin, 1924.
[9] F. W. J. OLVER, *Uniform, exponentially improved, asymptotic expansions for the generalized exponential integral*, SIAM J. Math. Anal. **22** (1991), 1460–1474.
[10] F. W. J. OLVER, *Uniform, exponentially improved, asymptotic expansions for the Confluent Hypergeometric function and other integral transforms*, SIAM J. Math. Anal. **22** (1991), 1475–1489.
[11] F. W. J. OLVER, *Asymptotics and Special Functions*, A. K. Peters Ltd., Wellesley, 1997.
[12] F. W. J. OLVER, D. W. LOZIER, R. F. BOISVERT AND C. W. CLARK (EDS.), *NIST Handbook of Mathematical Functions*, Cambridge University Press, New York, 2010.
[13] R. B. PARIS AND D. KAMINSKI, *Asymptotics and Mellin–Barnes Integrals*, Cambridge University Press, 2001.