A LOGARITHMIC MEAN AND INTERSECTIONS
OF OSCULATING HYPERPLANES IN R^n

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Abstract. We discuss a special case of the means defined in [1]. Let C be the curve in R^n with vector equation $\hat{\alpha}(t) = \langle t, t \log t, \ldots, t(\log t)^{n-1} \rangle$. Let $0 < a_1 < \cdots < a_n$ and let O_k be the osculating hyperplane to C at a_k. Then we show that $O_1, \ldots, O_n$ have a unique point of intersection, $P = (i_1, \ldots, i_n) \in R^n$, and in particular, i_1 equals the mean

$$M(a_1, \ldots, a_n) = (n-1)! \sum_{j=1}^{n} \frac{a_j}{\prod_{\substack{i=1 \\ i \neq j}}^{n} (\ln a_j - \ln a_i)},$$

the logarithmic mean of Neuman.

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