

THE RESURGENCE PROPERTIES OF THE LARGE ORDER ASYMPTOTICS OF THE ANGER—WEBER FUNCTION II

GERGÓ NEMES

Abstract. In this paper, we derive a new representation for the Anger–Weber function, employing the reformulation of the method of steepest descents by C. J. Howls (Howls, Proc. R. Soc. Lond. A **439** (1992) 373–396). As a consequence of this representation, we deduce a number of properties of the large order asymptotic expansion of the Anger–Weber function, including explicit and realistic error bounds, asymptotic approximations for the late coefficients, exponentially improved asymptotic expansions, and the smooth transition of the Stokes discontinuities.

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