AN ASYMPTOTIC EXPANSION FOR THE GENERALISED QUADRATIC GAUSS SUM REVISITED

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Abstract. An asymptotic expansion for the generalised quadratic Gauss sum

\[ S_N(x, \theta) = \sum_{j=1}^{N} \exp(\pi i x j^2 + 2\pi i j \theta), \]

where \( x, \theta \) are real and \( N \) is a positive integer, is obtained as \( x \to 0 \) and \( N \to \infty \) such that \( Nx \) is finite. The form of this expansion holds for all values of \( Nx + \theta \) and, in particular, in the neighbourhood of integer values of \( Nx + \theta \). A simple bound for the remainder in the expansion is derived. Numerical results are presented to demonstrate the accuracy of the expansion and the sharpness of the bound.


Keywords and phrases: Quadratic Gauss sum, asymptotics, curlicues.

REFERENCES