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PROBABILITY DISTRIBUTIONS OF EXTREMES OF SELF–SIMILAR GAUSSIAN RANDOM FIELDS

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Abstract. We have obtained some upper bounds for probability distributions of extremes of a self-similar Gaussian random field with stationary rectangular increments, which is defined on a compact space. In the paper we also present the probability distributions of extremes for normalized self-similar Gaussian random fields with stationary rectangular increments defined on $\mathbb{R}^2_+$. In our work we have used the techniques developed for self-similar fields and based on the classical series analysis of the supremum distribution for Gaussian fields.

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