

THE NECESSARY AND SUFFICIENT CONDITIONS FOR GENERAL HADAMARD PRODUCT OF CLASSES OF ANALYTIC FUNCTIONS

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Abstract. Let $P_a(A, B)$ be the classes of analytic functions $f(z)$, where $f(z) \prec \frac{a+Az}{1-Bz}$, $A + aB \neq 0$ and $|B| \leq 1$. For classes $\mathcal{H}_1, \mathcal{H}_2, \dots, \mathcal{H}_n$ of analytic functions, we define the general hadamard product of the form $\mathcal{H}_1 *_{m_1} \mathcal{H}_2 *_{m_2} \mathcal{H}_3 *_{m_3} \dots *_{m_{n-1}} \mathcal{H}_n(z) = \{f_1 *_{m_1} f_2 *_{m_2} f_3 *_{m_3} \dots *_{m_{n-1}} f_n(z) : f_i \in \mathcal{H}_i, i = 1, 2, \dots, n, n \in \mathbb{Z}^+, m_i \in \mathbb{C}\}$. In this paper, we discuss the conditions for equality $P_{a_1}(A_1, B_1) *_{m_1} P_{a_2}(A_2, B_2) *_{m_2} \dots *_{m_{n-1}} P_{a_n}(A_n, B_n) = P_c(X, Y)$. Some consequences of the main results for known classes of analytic functions are also pointed out.

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