MAPPING PROPERTIES OF BASIC HYPERGEOMETRIC FUNCTIONS

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Abstract. It is known that the ratio of Gaussian hypergeometric functions can be represented by means of g-fractions. In this work, the ratio of q-hypergeometric functions are represented by means of g-fractions that lead to certain results on q-starlikeness of the q-hypergeometric functions defined on the open unit disk. Corresponding results for the q-convex case are also obtained.

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