SOLUTIONS TO FRACTIONAL
DIFFERENTIAL EQUATIONS THAT EXTEND

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Abstract. This note discusses the question: *When do nonlinear fractional differential equations of arbitrary order have solutions that extend to a maximal interval of existence?* We show that a growth condition on the right-hand side of the equation ensures that solutions will extend. The method uses a classical approach from analysis, namely the divergence of an infinite series. It is interesting to note that the growth condition is related to the order of the fractional differential equation involved. A YouTube video from the author that is designed to complement this research is available at `http://tinyurl.com/Extend-FDE`.

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