MEANS AND NONREAL INTERSECTION POINTS OF TAYLOR POLYNOMIALS

ALAN HORWITZ

Abstract. Suppose that \( f \in C^{r+1}(0, \infty) \), and let \( P_c \) denote the Taylor polynomial to \( f \) of order \( r \) at \( x = c \in [a,b] \). In [2] it was shown that if \( r \) is an odd whole number and \( f^{(r+1)}(x) \neq 0 \) on \([a,b]\), then there is a unique \( x_0, a < x_0 < b \), such that \( P_a(x_0) = P_b(x_0) \). This defines a mean \( M_r^f(a,b) \equiv x_0 \). In this paper we discuss the real parts of the pairs of complex conjugate nonreal roots of \( P_b - P_a \). We prove some results for \( r \) in general, but our most significant results are for the case \( r = 3 \). We prove in that case that if \( f(z) = z^p \), where \( p \) is an integer, \( p \notin \{0,1,2,3\} \), then \( P_b - P_a \) has nonreal roots \( x_1 \pm iy_1 \), with \( a < x_1 < b \) for any \( 0 < a < b \). This defines the countable family of means \( M_3^{zp}(a,b) \), where \( p = n \in \mathbb{Z} - \{0,1,2,3\} \). We construct a cubic polynomial, \( g \), whose real root gives the real part of the pair of complex conjugate nonreal roots of \( P_b - P_a \). Instead of working directly with a formula for the roots of a cubic, we use the Intermediate Value Theorem to show that \( g \) has a root in \((a,b)\).


Keywords and phrases: Mean, Taylor polynomial, nonreal roots.

REFERENCES