

SOME INEQUALITIES FOR THE VOLUME OF THE UNIT BALL

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Abstract. In the paper, the authors establish some new inequalities involving the volume of the unit ball in \mathbb{R}^n and refine some results of Alzer.

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