

ON THE ASYMPTOTICS OF A SEQUENCE OF LACUNARY BINOMIAL-TYPE POLYNOMIALS

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Abstract. We examine the asymptotics of a sequence of lacunary binomial-type polynomials $\rho_n(z)$ as $n \rightarrow \infty$ that have arisen in the problem of the expected number of independent sets of vertices of finite simple graphs. We extend the recent analysis of Gawronski and Neuschel by employing the method of steepest descents applied to an integral representation. The case of complex z with $|z| < 1$ is also considered. Numerical results are presented to illustrate the accuracy of the resulting expansions.

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REFERENCES

- [1] J. BROWN, K. DILCHER AND D. MANNA, *Asymptotics of a sequence of sparse binomial-type polynomials*, *Analysis* **32** (2012) 231–245.
- [2] R. B. DINGLE, *Asymptotic Expansions: Their Derivation and Interpretation*, Academic Press, London, 1973.
- [3] W. GAWRONSKI AND T. NEUSCHEL, *On a conjecture on sparse binomial-type polynomials by Brown, Dilcher and Manna*, *Analysis and Applications*, **12** (5) (2014) 511–522.
- [4] F. W. J. OLVER, D. W. LOZIER, R. F. BOISVERT AND C. W. CLARK (eds.), *NIST Handbook of Mathematical Functions*, Cambridge University Press, Cambridge, 2010.
- [5] R. B. PARIS, *Hadamard Expansions and Hyperasymptotic Evaluation: An Extension of the Method of Steepest Descents*, Cambridge University Press, Cambridge, 2011.