

ON MAXIMAL AREA INTEGRAL PROBLEM FOR ANALYTIC FUNCTIONS IN THE STARLIKE FAMILY

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Abstract. For an analytic function f defined on the unit disk $|z| < 1$, let $\Delta(r, f)$ denote the area of the image of the subdisk $|z| < r$ under f , where $0 < r \leq 1$. In 1990, Yamashita conjectured that $\Delta(r, z/f) \leq \pi r^2$ for convex functions f and it was finally settled in 2013 by Obradović and et. al. In this paper, we consider a class of analytic functions in the unit disk satisfying the subordination relation $zf'(z)/f(z) \prec (1 + (1 - 2\beta)\alpha z)/(1 - \alpha z)$ for $0 \leq \beta < 1$ and $0 < \alpha \leq 1$. We prove Yamashita's conjecture problem for functions in this class, which provides a partial solution to an open problem posed by Ponnusamy and Wirths.

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