EXPLICIT FORMULA FOR GENERALIZATION OF POLY–BERNOULLI NUMBERS AND POLYNOMIALS WITH a,b,c PARAMETERS

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Abstract. In this paper we investigate special generalized Bernoulli polynomials with a,b,c parameters that generalize classical Bernoulli numbers and polynomials. The present paper deals with some recurrence formulae for the generalization of poly-Bernoulli numbers and polynomials with a,b,c parameters. Poly-Bernoulli numbers satisfy certain recurrence relationships which are used in many computations involving poly-Bernoulli numbers. Obtaining a closed formula for generalization of poly-Bernoulli numbers with a,b,c parameters therefore seems to be a natural and important problem. By using the generalization of poly-Bernoulli polynomials with a,b,c parameters of negative index we define symmetrized generalization of poly-Bernoulli polynomials with a; b; c parameters of two variables and we prove duality property for them. Also by Stirling numbers of the second kind we will find a closed formula for them. Furthermore we generalize the Arakawa-Kaneko Zeta functions and by using the Laplace-Mellin integral, define generalization of Arakawa-Kaneko Zeta functions with a,b,c parameters and obtain an interpolation formula for the generalization of poly- Bernoulli numbers and polynomials with a,b,c parameters. Furthermore we present a link between this type of Zeta functions and Dirichlet series. By our interpolation formula, we will interpolate the generalization of Arakawa-Kaneko Zeta functions with a,b,c parameters.

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