

SIDELNIKOV INEQUALITY AND LEGENDRE POLYNOMIALS

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Abstract. Extremal arrangements (in various meanings of “extremal”) of points on a sphere have long been a subject of much interest (see [3] for a survey of early works).

One class of such extremal arrangements, consisting of “semidesigns” introduced in [4], is related to so-called Sidelnikov inequality [8]. In this paper we present a very simple proof of a discrete case of Sidelnikov inequality and show that it becomes equality on semidesigns.

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