

ESTIMATION FOR INITIAL COEFFICIENTS OF BI-UNIVALENT λ -CONVEX ANALYTIC FUNCTIONS IN THE UNIT DISC

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Abstract. In this paper, we find improved bounds on the moduli of the second and third coefficients of the Taylor-Maclaurin's series for bi-univalent λ -convex analytic functions in the unit disc of the complex plane. We also find an estimate on modulus of the fourth coefficient. The result on the fourth coefficient is new. Our bounds are obtained by refining well known estimates for the initial coefficients of the Carthéodory functions.

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