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APPROXIMATIONS USING HILBERT TRANSFORM OF WAVELETS

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Abstract. Hilbert transform of wavelets has been used to approximate functions in $L^2(\mathbb{R})$. It is proved that Hilbert transform of wavelets with many vanishing moments does a good job in approximating smooth functions in $L^2(\mathbb{R})$. We also prove that Hölder continuity of a function helps in the decay of wavelet coefficients and thereby helps in approximating it. Finally, we give a result that relates the Hilbert transform of wavelet with dyadic scale differential operator and use it to decrease the wavelet coefficients.

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