

INTEGRAL REPRESENTATIONS OF PRODUCTS OF AIRY FUNCTIONS RELATED TO FRACTIONAL CALCULUS

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Abstract. In this paper, using the inverse Laplace transform of multi-valued functions we show new identities for the powers of Airy functions in terms of the Riemann-Liouville and Weyl fractional integrals of order α . In this sense, we get new integral representations for the special functions including trigonometric functions in terms of the M-Wright function and Hilbert transform. Also, we get the Hadamard fractional integrals of Airy functions in terms of the Widder potential and Mellin transforms of the Volterra functions.

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