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SOME APPROXIMATION PROPERTIES OF HEXAGONAL FOURIER SERIES

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Abstract. L. Leindler, A. Meir and V. Totik considered the φ-norm on $C_{2\pi}$ (the space 2π-periodic continuous functions) and estimated the deviation $\|A_n(f)-f\|_{\varphi}$ in terms of the modulus of continuity of $f \in C_{2\pi}$, where (A_n) is a sequence of convolution operators from $C_{2\pi}$ into itself and φ is an increasing function on $(0,\infty)$ (Acta Math. Hung. 45 (1985), 441-443). In the present paper, an analogue of the theorem of Leindler, Meir and Totik is proved for functions periodic with respect to the hexagon lattice. Also, this theorem is applied to obtain estimates for approximation by partial sums of hexagonal Fourier series in Hölder and generalized Hölder norms.

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