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POINTWISE ESTIMATE FOR LINEAR COMBINATIONS OF PHILLIPS OPERATORS

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Abstract. For pointwise approximation of bounded continuous functions by linear combinations of Phillips operators we represent equivalent relation by means of Ditzian-Totik modulus of smoothness. The rate of approximation is better compared with similar estimates, proved in the past for other Szász-type operators.

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Paper JCA-08-03

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