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ON THE LIMIT INFERIOR AND LIMIT SUPERIOR FOR DOUBLE SEQUENCES

ÜMIT TOTUR

Abstract. Let (u_{mn}) be a double sequence of real numbers such that $\limsup \sigma_{mn}(u) = \beta$ and $\liminf \sigma_{mn}(u) = \alpha$, where $\sigma_{mn}(u) = \frac{1}{(m+1)(n+1)} \sum_{j=0}^{m} \sum_{k=0}^{n} u_{jk}$, and $\alpha \neq \beta$. In this paper, it is presented that $\limsup u_{mn} = \beta$ and $\liminf u_{mn} = \alpha$ if the following conditions hold: For $\lambda > 1$

$$\liminf \frac{1}{([\lambda m]-m)([\lambda n]-n)} \sum_{j=m+1}^{[\lambda m]} \sum_{k=n+1}^{[\lambda n]} (u_{jk} - u_{mn}) \geqslant 2(\beta-\alpha)\frac{\lambda(2\lambda-1)}{(\lambda-1)^2},$$

for $0 < \lambda < 1$

$$\liminf \frac{1}{(m-[\lambda m])(n-[\lambda n])} \sum_{j=[\lambda m]+1}^{m} \sum_{k=[\lambda n]+1}^{n} (u_{mn} - u_{jk}) \geqslant 2(\beta-\alpha)\frac{\lambda}{(\lambda-1)^2},$$

where $[\lambda n]$ denotes the integer part of λn.

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