Journal of Classical Analysis
Volume 8, Number 1 (2016), 59–64
doi:10.7153/jca-08-05

INEQUALITIES INVOLVING THE INTEGRALS OF POLYNOMIALS AND THEIR POLAR DERIVATIVES

N. A. RATHER, SUHAIL GULZAR AND S. H. AHANGAR

Abstract. For the class of polynomials $P(z)$ of degree n having all their zeros in $|z| \leqslant k$ where $k \leqslant 1$, Aziz [1] proved that for each $q > 0$,

$$n\left\{\int_0^{2\pi}\left|\frac{P(e^{i\theta})}{P'(e^{i\theta})}\right|^q d\theta\right\}^{1/q} \leqslant \left\{\int_0^{2\pi}\left|k+e^{i\theta}\right|^q d\theta\right\}^{1/q}.$$

In this paper, we extend this inequality to the polar derivative in the sense that we take the polar derivative $D_\alpha P(z)$ in place of ordinary derivative $P'(z)$ of polynomial $P(z)$. We also obtain analogous inequalities for the class of lacunary polynomials $P(z) = a_n z^n + \sum_{j=\mu}^{n} a_{n-j} z^{n-j}$, $1 \leqslant \mu \leqslant n$, of degree n having all their zeros in $|z| \leqslant k$, $k \leqslant 1$.

Mathematics subject classification (2010): 30C10, 30A10, 41A17.
Keywords and phrases: Polynomials, inequalities in the complex domain, polar derivative.

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Paper JCA-08-05