

A NOTE ON DEGENERATE HERMITE POLY-BERNOULLI NUMBERS AND POLYNOMIALS

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Abstract. In this paper, we introduce a new class of degenerate Hermite poly-Bernoulli polynomials and give some identities of these polynomials related to the Stirling numbers of the second kind. Some implicit summation formulae and general symmetry identities are derived by using different analytical means and applying generating functions. These results extend some known summations and identities of degenerate Hermite poly-Bernoulli numbers and polynomials.

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