

SUMS OF QUADRATIC HALF INTEGER HARMONIC NUMBERS OF ALTERNATING TYPE

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Abstract. Half integer values of quadratic harmonic numbers and reciprocal binomial coefficients sums are investigated in this paper. Closed form representations of double integral expressions are developed in terms of special functions.

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