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HARMONIC SERIES WITH POLYGAMMA FUNCTIONS

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Abstract. The paper is about evaluating in closed form the following classes of series involving the product of the nth harmonic number and the polygamma functions

$$S_k = \sum_{n=1}^{\infty} H_n \left(\zeta(k) - 1 - \frac{1}{2^k} - \cdots - \frac{1}{n^k} \right) = \frac{(-1)^k}{(k-1)!} \sum_{n=1}^{\infty} H_n \psi^{(k-1)}(n+1), \quad k \geqslant 3,$$

$$T_k = \sum_{n=1}^{\infty} nH_n \left(\zeta(k) - 1 - \frac{1}{2^k} - \cdots - \frac{1}{n^k} \right) = \frac{(-1)^k}{(k-1)!} \sum_{n=1}^{\infty} nH_n \psi^{(k-1)}(n+1), \quad k \geqslant 4,$$

and

$$R_k = \sum_{n=1}^{\infty} H_n^2 \left(\zeta(k) - 1 - \frac{1}{2^k} - \cdots - \frac{1}{n^k} \right) = \frac{(-1)^k}{(k-1)!} \sum_{n=1}^{\infty} H_n^2 \psi^{(k-1)}(n+1), \quad k \geqslant 3,$$

where k is an integer.

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REFERENCES

[1] D. D. BONAR AND M. J. KOURY, *Real Infinite Series*, MAA, Washington DC, 2006.
[2] D. BORWEIN AND J. M. BORWEIN, *On an intriguing integral and some series related to* $\zeta(4)$, Proc. Amer. Math. Soc., **123** (1995), 1191–1198.
[3] D. BORWEIN, J. M. BORWEIN AND R. GIRGENSOHN, *Explicit Evaluation of Euler Sums*, Proc. Edinburgh Math. Soc., **38** (1995), 277–294.
[4] J. CHOI AND H. M. SRIVASTAVA, *Explicit Evaluations of Euler and Related Sums*, Ramanujan J., **10** (2005), 51–70.
[5] O. FURDUI, *Limits, Series and Fractional Part Integrals. Problems in Mathematical Analysis*, Springer, New York, 2013.
[6] O. FURDUI, *Series involving products of two harmonic numbers*, Math. Mag., **84** (2011), 371–377.
[7] H. M. SRIVASTAVA AND J. CHOI, *Series Associated with the Zeta and Related Functions*, Kluwer Academic Publishers, Dordrecht, 2001.
[8] H. M. SRIVASTAVA AND J. CHOI, *Zeta and* q*-Zeta Functions And Associated Series And Integrals*, Elsevier, Amsterdam, 2012.
[9] A. SOFO AND D. CVIJOVIĆ, *Extensions of Euler harmonic sums*, Appl. Anal. Discrete Math., **6** (2012), 317–328.
[10] E. T. WHITTAKER AND G. N. WATSON, *A Course of Modern Analysis*, Fourth Edition, Cambridge, AT The University Press, 1927.