EXTENSION OF WEYL–HEISENBERG WAVE PACKET BESSEL SEQUENCES TO DUAL FRAMES IN $L^2(\mathbb{R})$

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Abstract. Christensen et al. proved in [Extensions of Bessel sequences to dual pairs of frames, Appl. Comput. Harmon. Anal., 34 (2013), 224–233] that in any separable Hilbert space, any pairs of Bessel sequences (even if the given Bessel system is Gabor system in $L^2(\mathbb{R})$) can be extended to a pair of dual frames. In this paper, we extend results by Christensen et al. to a pair of Bessel sequences in $L^2(\mathbb{R})$ having wave packet structure to a pair of dual frames such that extension have wave packet structure as well. We present sufficient conditions for the extension of a pair of Bessel sequences to wave packet type dual frames for $L^2(\mathbb{R})$. Several examples and counter-examples are given to illustrate our results.

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