A NEW PROOF FOR A CLASSICAL QUADRATIC HARMONIC SERIES

CORNEL IOAN VĂLEAN

Abstract. In the following paper we intend to present a new way of calculating a series similar to the quadratic series of Au-Yeung (see [1])

\[ \sum_{n=1}^{\infty} \frac{H_n^2}{n^3} = \frac{7}{2} \zeta(5) - \zeta(2)\zeta(3), \]

where \( H_n \) denotes the \( n \)th harmonic number. We will prove the result by combining a series of techniques based on the calculation of two special logarithmic integrals, the elementary manipulations of series and then the use of the Euler’s identity in (1).


Keywords and phrases: Logarithmic integrals, harmonic numbers, quadratic series, Euler sums, Riemann zeta function.

REFERENCES