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A NEW PROOF FOR A CLASSICAL QUADRATIC HARMONIC SERIES

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Abstract. In the following paper we intend to present a new way of calculating a series similar to the quadratic series of Au-Yeung (see [1])

$$\sum_{n=1}^{\infty} \frac{H_n^2}{n^3} = \frac{7}{2}\zeta(5) - \zeta(2)\zeta(3),$$

where H_n denotes the nth harmonic number. We will prove the result by combining a series of techniques based on the calculation of two special logarithmic integrals, the elementary manipulations of series and then the use of the Euler's identity in (1).

Mathematics subject classification (2010): 40C10, 40A05.
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