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APPROXIMATION OF PERIODIC FUNCTIONS BY ZYGMUND MEANS IN ORLICZ SPACES

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Abstract. In the present work we investigate the approximation of the functions by Zygmund means in the Orlicz spaces $L_M(\mathbb{T})$ in the terms of the best approximation $E_n(f)_M$ and modulus of smoothness $\omega_k(\cdot,f)_M$.

Mathematics subject classification (2010): 41A10, 41A25, 42A10, 42B25, 46E30.
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