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ON THE RATIO OF TWO SETS IN REAL LINE

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Abstract. In this paper, assuming Martin's axiom we show that there exists a Lebesgue measurable subset A of the real line $\mathbb{R}$ such that the set $\{c \in \mathbb{R} : R(A, c+A)$ contains an interval$\}$ is non-measurable. Here the set $R(A,c+A) = \{\frac{a}{c+a'} : a, a' \in A, c+a' \neq 0\}$. Also other two results on the ratio set of linear sets are presented.

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