

BOUNDARY ASYMPTOTICS OF THE RELATIVE BERGMAN KERNEL METRIC FOR ELLIPTIC CURVES III: 1 & ∞

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Abstract. Explicit asymptotic formulas of the relative Bergman kernel metric for a Legendre family of elliptic curves near the moduli space boundary points 1 and ∞ are obtained respectively. These behaviors also characterize the Poincaré hyperbolic metric and its Kähler potential on $\mathbb{C} \setminus \{0, 1\}$.

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