GENERALIZED STIELTJES CONSTANTS AND INTEGRALS INVOLVING THE LOG–LOG FUNCTION: KUMMER'S THEOREM IN ACTION

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Abstract. In this note, we recall Kummer's Fourier series expansion of the 1-periodic function that coincides with the logarithm of the Gamma function on the unit interval $(0,1)$, and we use it to find closed forms for some numerical series related to the generalized Stieltjes constants, and some integrals involving the function $x \mapsto \ln\ln(1/x)$.

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