NEW RESULTS CONTAINING QUADRATIC HARMONIC NUMBERS

ANTHONY SOFO

Abstract. In this paper we give a combinatorial proof of the quadratic harmonic series $\sum_{n \geqslant 1} \frac{H_n^2}{n^{2q+1}}$ in terms of zeta functions and then extend the result to express $\sum_{n \geqslant 1} \frac{H_n^2}{(n+r)^{2q+1}}, (q,r) \in \mathbb{N}$, in closed form in terms of zeta functions.

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