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HOMOGENEOUS BETA–TYPE FUNCTIONS

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Abstract. All beta-type functions, i.e. the functions $B_f : (0,\infty)^2 \to (0,\infty)$ of the form

$$B_f(x,y) = \frac{f(x)f(y)}{f(x+y)}$$

for some $f : (0,\infty) \to (0,\infty)$, which are p-homogeneous, are determined. Applying this result, we show that a beta-type function is a homogeneous mean iff it is the harmonic one. A reformulation of a result due to Heuvers in terms of a Cauchy difference and the harmonic mean is given.

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