

ON AN INTEGRAL INEQUALITY OF M. A. MALIK

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Abstract. In this paper, we shall prove some L^r inequalities for the polar derivative of a polynomial having zeros in $|z| \leq k \leq 1$ and thereby obtain generalizations and refinements of an integral inequality due to Malik [16]. Besides, we shall also provide an alternative proof of a result due to Dewan et al. [9] which is independent of Laguerre's theorem.

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REFERENCES

- [1] A. AZIZ, *Integral mean estimates for polynomials with restricted zeros*, J. Approx. Theory, **55** (1988), 232–239.
- [2] A. AZIZ AND Q. M. DAWOOD, *Inequalities for a polynomial and its derivative*, J. Approx. Theory, **54** (1988), 306–313.
- [3] A. AZIZ AND N. A. RATHER, *Some Zygmund type inequalities for polynomials*, J. Math. Anal. Appl., **289** (2004), 14–29.
- [4] A. AZIZ AND N. A. RATHER, *A refinement of a theorem of Paul Turán concerning polynomials*, Math. Ineq. Appl., **1** (1998), 231–238.
- [5] A. AZIZ AND W. M. SHAH, *An integral mean estimate for polynomials*, Indian J. Pure Appl. Math., **28** (1997), 1413–1419.
- [6] S. BERNSTEIN, *Sur é ordre de la meilleure approximation des fonctions continues par des polynomes de degré donné*, Mem. Acad. R. Belg., **4** (1912), 1–103.
- [7] M. BDKHAM AND H. A. SOLEIMAN MEZERJI, *Some inequalities for the polar derivative of polynomials in complex domain*, Complex Anal. Oper. Theory, (2013), 1257–1266.
- [8] K. K. DEWAN, N. SINGH AND R. LAL, *Inequalities for the polar derivative of a polynomial*, Int. J. Pure. Appl. Math., **33** (2006), 109–117.
- [9] K. K. DEWAN, N. SINGH AND A. MIR, *Extensions of some polynomial inequalities to the polar derivative*, J. Math. Anal. Appl., **352** (2009), 807–815.
- [10] K. K. DEWAN, A. MIR AND R. S. YADAV, *Integral mean estimates for polynomials whose zeros are with in a circle*, Int. J. Math. Math. Sci., **4** (2001), 231–235.
- [11] K. K. DEWAN, N. SINGH, A. MIR AND A. BHAT, *Some inequalities for the polar derivative of a polynomial*, Southeast Asian Bull. Math., **34** (2010), 69–77.
- [12] N. K. GOVIL, *Some inequalities for derivative of polynomials*, J. Approx. Theory, **66** (1991), 29–35.
- [13] N. K. GOVIL, Q. I. RAHMAN AND G. SCHMEISSER, *On the derivative of a polynomial*, Illinois J. Math., **23** (1979), 319–330.
- [14] E. HILLE, *Analytic function theory*, Vol II, Ginn and Company, New York, Toronto, 1962.
- [15] M. A. MALIK, *On the derivative of a polynomial*, J. London Math. Soc., **1** (1969), 57–60.
- [16] M. A. MALIK, *An integral mean estimates for polynomials*, Proc. Amer. Math. Soc., **91** (1984), 281–284.
- [17] A. MIR AND B. A. DAR, *On the polar derivative of a polynomial*, J. Ramanujan Math. Soc., **29** (2014), 403–412.
- [18] A. MIR, B. A. DAR AND Q. M. DAWOOD, *Some integral mean estimates for polynomials*, Int. Math. Forum, **8** (2013), 501–511.
- [19] N. A. RATHER, SUHAIL GULZAR AND S. H. AHANGAR, *Inequalities involving the integrals of polynomials and their polar derivatives*, Jour. Class. Anal., **8** (2016), 59–64.

- [20] P. TURÁN, *Über die Ableitung von Polynomen*, *Compositio Math.*, **7** (1939), 89–95.
- [21] A. ZIREH, *On the polar derivative of a polynomial*, *Bull. Iranian Math. Soc.*, **40** (2014), 967–976.