ON AN INTEGRAL INEQUALITY OF M. A. MALIK

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Abstract. In this paper, we shall prove some L^r inequalities for the polar derivative of a polynomial having zeros in $|z| \leqslant k \leqslant 1$ and thereby obtain generalizations and refinements of an integral inequality due to Malik [16]. Besides, we shall also provide an alternative proof of a result due to Dewan et al. [9] which is independent of Laguerre's theorem.

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