Journal of Classical Analysis
Volume 10, Number 2 (2017), 131–146
doi:10.7153/jca-10-13

# CLOSURE OF THE LINEAR SPAN OF AN EXPONENTIAL SYSTEM IN A WEIGHTED BANACH SPACE

ELIAS ZIKKOS

*Abstract.* For a certain class of sequences with repeated terms,

$$\{\lambda_n,\mu_n\}_{n=1}^{\infty} := \{\underbrace{\lambda_1,\lambda_1,\ldots,\lambda_1}_{\mu_1 \text{ times}},\underbrace{\lambda_2,\lambda_2,\ldots,\lambda_2}_{\mu_2 \text{ times}},\ldots,\underbrace{\lambda_k,\lambda_k,\ldots,\lambda_k}_{\mu_k \text{ times}},\ldots\},$$

we prove that every function belonging to the closed span of the exponential system

$$\{x^k e^{\lambda_n x} : n\in\mathbb{N},\ k=0,1,2,\ldots,\mu_n-1\},$$

in some weighted Banach spaces on the real line, extends analytically as an entire function by admitting a series representation of the form

$$\sum_{n=1}^{\infty}\left(\sum_{k=0}^{\mu_n-1} c_{n,k}z^k\right)e^{\lambda_n z},\quad c_{n,k}\in\mathbb{C},\quad \forall\, z\in\mathbb{C}.$$

*Mathematics subject classification* (2010): 30B50, 30B60, 46E15, 46E20.
*Keywords and phrases*: Completeness, closure, minimality, Taylor-Dirichlet series.

## REFERENCES

[1] J. M. ANDERSON, K. G. BINMORE, *Closure theorems with applications to entire functions with gaps*, Trans. Amer. Math. Soc. **161** (1971) 381–400.

[2] G. T. DENG, *Incompleteness and closure of a linear span of exponential system in a weighted Banach space*, J. Approx. Theory **125** no. 1 (2003), 1–9.

[3] W. H. J. FUCHS, *On the closure of* $\{e^{-t}t^{a_n}\}$, Proc. Cambridge Philos. Soc. **42** (1946), 91–105.

[4] P. MALLIAVIN, *Sur quelques procédés d'extrapolation*, Acta Math. **93** (1955) 179–255.

[5] J. NING, G. T. DENG, C. YI, *Incompleteness and closure of the multiplicity system* $\{t^k e^{\lambda_j t}\}$ *in the weighted Banach space*, J. Math. Anal. Appl. **341** no. 2 (2008), 1007–1017.

[6] X. YANG, *Incompleteness of exponential system in the weighted Banach space*, J. Approx. Theory **153** no. 1 (2008), 73–79.

[7] E. ZIKKOS, *Completeness of an exponential system in weighted Banach spaces and closure of its linear span*, J. Approx. Theory **146** no. 1 (2007), 115–148.