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VECTOR–STABILITY OF MULTIPLE VECTOR REFINABLE VECTORS

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Abstract. The stability is an expected property for refinable vectors, which is widely considered in the study of refinement equations. There are two types of stability for refinable vectors. One is *the ordinary-stability*, the other is *the vector-stability*. *The ordinary-stability* considers the stability of entries of refinable vectors, but *the vector-stability* considers the stability of refinable vectors themselves where they are considered as elements of super Hilbert spaces. In this paper, we give a necessary and sufficient condition for refinable vectors to be vector-stable. Our results improve some known ones.

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