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THE POWER COLLECTION METHOD FOR CONNECTION RELATIONS: MEIXNER POLYNOMIALS

MICHAEL A. BAEDER, HOWARD S. COHL, ROBERTO S. COSTAS-SANTOS AND WENQING XU

Abstract. We introduce the power collection method for easily deriving connection relations for certain hypergeometric orthogonal polynomials in the (q-)Askey scheme. We summarize the full-extent to which the power collection method may be used. As an example, we use the power collection method to derive connection and connection-type relations for Meixner and Krawtchouk polynomials. These relations are then used to derive generalizations of generating functions for these orthogonal polynomials. The coefficients of these generalized generating functions are in general, given in terms of multiple hypergeometric functions. From derived generalized generating functions, we deduce corresponding contour integral and infinite series expressions by using orthogonality.

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