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A NEW APPROACH TO STEINER SYMMETRIZATION OF COERCIVE CONVEX FUNCTIONS

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Abstract. A new way of defining Steiner symmetrization of coercive convex functions is proposed which does not use the Steiner symmetrization of level sets. Some fundamental properties of the new Steiner symmetrization are proved.

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