

EVALUATION OF APÉRY-LIKE SERIES THROUGH MULTISECTION METHOD

WENCHANG CHU AND FLAVIA LUCIA ESPOSITO

Abstract. By combining the multisection series method with the power series expansion of arcsin-function, we investigate Apéry-like infinite series involving the central binomial coefficients in denominators. By constructing and resolving systems of linear equations, numerous remarkable infinite series formulae (generated by using an appropriate computer algebra system) for π and special values of the logarithm function are established, including some recent results due to Almkvist *et al.* (2003) and Zheng (2008).

Mathematics subject classification (2010): Primary 11Y60, Secondary 14G10.

Keywords and phrases: Multisection series, Apéry-like series, central binomial coefficient.

REFERENCES

- [1] G. ALMKVIST *et al.*, *Some new formulas for π* , *Experimental Mathematics* **12** (2003), 441–456.
- [2] R. APÉRY, *Irrationalité de $\zeta(2)$ et $\zeta(3)$* , *Journées Arithmétiques de Luminy: Astérisque* **61** (1979), 11–13.
- [3] D. H. BAILEY *et al.*, *The quest for π* , *Mathematical Intelligencer* **19**:1 (1997), 50–57; MR1439159 (98b:01045).
- [4] N. D. BARUAH *et al.*, *Ramanujan's series for $1/\pi$: a survey*, *American Mathematical Monthly* **109**:7 (2009), 567–587.
- [5] J. M. BORWEIN *et al.*, *Central binomial sums, multiple Clausen values and zeta values*, *Experimental Mathematics* **10** (2001), 25–34.
- [6] J. M. BORWEIN AND R. GIRGENSOHN, *Evaluations of binomial series*, *Aequationes Mathematicae* **70** (2005), 25–36.
- [7] D. BRADLEY, *More Apéry-like formulae: On representing values of the Riemann zeta function by infinite series damped by central binomial coefficients*, preprint, 2002; <http://www.math.umaine.edu/~bradley/papers/bivar5.pdf>.
- [8] W. CHU, *Hypergeometric series and the Riemann zeta function*, *Acta Arithmetica* **82** (1997), 103–118.
- [9] W. CHU, *Symmetric functions and the Riemann Zeta series*, *Indian Journal of Pure and Applied Mathematics* **31**:12 (2000), 1677–1689; MR2002e:11112.
- [10] W. CHU, *Dougall's bilateral ${}_2H_2$ -series and Ramanujan-like π -formulae*, *Mathematics of Computation* **80**:276 (2011), 2223–2251.
- [11] W. CHU AND D. Y. ZHENG, *Infinite series with harmonic numbers and central binomial coefficients*, *International Journal of Number Theory* **5**:3 (2009), 429–448.
- [12] L. COMTET, *Advanced Combinatorics*, Reidel, Boston, Massachusetts, 1974.
- [13] C. ELSNER, *On sums with binomial coefficient*, *Fibonacci Quarterly* **43**:1 (2005), 31–45.
- [14] M. GENČEV, *Binomial sums involving harmonic numbers*, *Mathematica Slovaca* **61**:2 (2011), 215–226.
- [15] M. L. GLASSER, *A generalized Apéry series*, *Journal of Integer Sequences* **15** (2012), #Article 12.4.3.
- [16] J. GUILLERA, *Some binomial series obtained by the WZ-method*, *Advances in Applied Mathematics* **29**:4 (2002), 599–603.
- [17] J. GUILLERA, *Hypergeometric identities for 10 extended Ramanujan-type series*, *Ramanujan Journal* **15**:2 (2008), 219–234.

- [18] D. H. LEHMER, *Interesting series involving the central binomial coefficient*, American Mathematical Monthly **92** (1985), 449–457.
- [19] H. MUZAFFAR, *Some interesting series arising from the power series expansion of $(\sin^{-1}x)^q$* , International Journal of Mathematics and Mathematical Sciences **14** (2005), 2329–2336.
- [20] A. J. VAN DER POORTEN, *A proof that Euler missed...*, Mathematical Intelligencer **1** (1979), 195–203.
- [21] T. SHERMAN, *Summations of Glaisher and Apéry-like numbers*, available at <http://math.arizona.edu/~ura/001/sherman.travis/series.pdf>.
- [22] L. ZHANG AND W. JI, *The series of reciprocals of non-central binomial coefficients*, American Journal of Computational Mathematics **3** (2013), 31–37.
- [23] B. SURY et al., *Identities involving reciprocals of binomial coefficients*, Journal of Integer Sequences **7** (2004), #Article 4.2.8.
- [24] D. Y. ZHENG, *Multisection method and further formulae for π* , Indian Journal of Pure and Applied Mathematics **39**:2 (2008), 137–155.
- [25] I. J. ZUCKER, *On the series $\sum_{k=1}^{\infty} \binom{2k}{k}^{-1} k^{-n}$* , Journal of Number Theory **20**:1 (1985), 92–102.