Journal of Classical Analysis
Volume 12, Number 2 (2018), 83–91
doi:10.7153/jca-2018-12-07

ON SHARPENING OF A THEOREM OF T. J. RIVLIN

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Abstract. Let $p(z)=a_0+a_1z+a_2z^2+a_3z^3+\cdots+a_nz^n$ be a polynomial of degree n. According to a well-known theorem of Rivlin [11], if $p(z)$ is a polynomial of degree n having no zeros inside the unit circle, then for $0<r\leqslant 1$,

$$\max_{|z|=r}|p(z)|\geqslant\left(\frac{r+1}{2}\right)^n\max_{|z|=1}|p(z)|.$$

In this paper, we generalize and sharpen the above result of Rivlin. Our result also sharpens a recently proved result of Govil and Nwaeze [3]. Also, we present some examples to show that in some cases the improvement obtained by our theorem can be considerably significant.

Mathematics subject classification (2010): 15A18, 30C10, 30C15, 30A10.
Keywords and phrases: Inequalities, polynomials, zeros.

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Paper JCA-12-07