

ON SHARPENING OF A THEOREM OF T. J. RIVLIN

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Abstract. Let $p(z) = a_0 + a_1z + a_2z^2 + a_3z^3 + \cdots + a_nz^n$ be a polynomial of degree n . According to a well-known theorem of Rivlin [11], if $p(z)$ is a polynomial of degree n having no zeros inside the unit circle, then for $0 < r \leq 1$,

$$\max_{|z|=r} |p(z)| \geq \left(\frac{r+1}{2}\right)^n \max_{|z|=1} |p(z)|.$$

In this paper, we generalize and sharpen the above result of Rivlin. Our result also sharpens a recently proved result of Govil and Nwaeze [3]. Also, we present some examples to show that in some cases the improvement obtained by our theorem can be considerably significant.

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