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ON COEFFICIENT FUNCTIONALS ASSOCIATED WITH THE ZALCMAN CONJECTURE

SARITA AGRAWAL AND SWADESH KUMAR SAHOO

Abstract. For a function f which is analytic and univalent in the unit disk $\{z \in \mathbb{C} : |z| < 1\}$ having the power series expansion of the normalized form $z + \sum_{n=2}^{\infty} a_n z^n$, Zalcman conjectured that $|a_n^2 - a_{2n-1}| \leqslant (n-1)^2$, $n = 2,3,\ldots$. In this article, we obtain the sharp estimate for the classical Zalcman coefficient functional $a_n^2 - a_{2n-1}$ for the above class of functions with the restriction that the n-th coefficient, a_n, has certain integral representation associated with probability measure. Moreover, we also study a similar problem for the classes of functions of the above form whose coefficients satisfy certain inequalities.

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