ON THE CONVERGENCE OF SERIES WITH RECURSIVELY DEFINED TERMS

N. S. HOANG

Abstract. We investigate the asymptotic behavior of a sequence \((x_n)_{n=0}^{\infty}\) defined recursively by \(x_{n+1} = f(x_n)\), \(n \geq 0\) where \(f : [0, \infty) \to [0, \infty)\) is a continuous function. A fundamental criterion on the function \(f(x)\) for estimating the rate of decay of \(x_n\) as \(n\) tends to \(\infty\) and for testing convergence of the series \(\sum_{n=0}^{\infty} x_n\) is proposed and justified. Criteria for testing absolute and conditional convergence of \(\sum_{n=0}^{\infty} x_n\) when \(f(x)\) is not a non-negative function are also formulated and proved.

Mathematics subject classification (2010): 40A05, 40F05.

Keywords and phrases: Convergence, series, rate of decay, comparison test.

REFERENCES