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A SHARP CARATHÉODORY'S INEQUALITY ON THE RIGHT HALF PLANE

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Abstract. In this paper, a boundary version of Carathéodory's inequality on the right half plane is investigated. Here, the function $Z(s)$, is given as $Z(s)=1+c_1(s-1)+c_2(s-1)^2+...$ be an analytic in the right half plane with $\Re Z(s)\leqslant A\ \ (A>1)$ for $\Re s\geqslant 0$. We derive inequalities for the modulus of $Z(s)$ function, $|Z'(0)|$, by assuming the $Z(s)$ function is also analytic at the boundary point $s=0$ on the imaginary axis and finally, the sharpness of these inequalities is proved.

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