

## FUNDAMENTAL THEOREMS OF SUMMABILITY THEORY FOR A NEW TYPE OF SUBSEQUENCES OF DOUBLE SEQUENCES

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*Abstract.* In 2000, the notion of a subsequence of a double sequence was introduced [3]. Using this definition, a multidimensional analogue to a result from H. Steinhaus, that states that for any regular matrix  $A$  there exists a sequence of zeros and ones that is not  $A$ -summable, was proved. Additionally, an analogue of a result of R. C. Buck that states that a sequence  $x$  is convergent if and only if there exists a regular matrix  $A$  that sums every subsequence of  $x$  was presented. However, this definition imposes a restrictive condition on the entries of the double sequence that can be considered for the subsequence. In this article, we introduce a less restrictive new definition of a subsequence. We denote them by  $\beta$ -subsequences of a double sequence and show that analogues to these two fundamental theorems of summability still hold for these new subsequences.

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