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BOUNDS ON COEFFICIENTS AND THIRD HANKEL DETERMINANT FOR A CLASS OF ANALYTIC FUNCTIONS RELATED WITH CERTAIN CONIC DOMAIN

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Abstract. In this paper, we obtain upper bounds on initial coefficients and third Hankel determinant

$$H_{3,1}(f)=\begin{vmatrix} a_1 & a_2 & a_3 \\ a_2 & a_3 & a_4 \\ a_3 & a_4 & a_5 \end{vmatrix}$$

of the coefficients of analytic function $f(z)=z+a_2z^2+a_3z^3+\cdots$, belonging to the class $\mathscr{S}^*(q)$ in the open unit disk $\mathbb{D}$, which satisfies the subordination condition that

$$zf'(z)/f(z)\prec q(z)\quad (z\in\mathbb{D}),$$

where $q(z)=\sqrt{1+z^2}+z$. Several results are presented exhibiting improvement in earlier work.

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