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SCHUR'S THEOREM FOR MODIFIED DISCRETE FOURIER TRANSFORM

N. O. KOTELINA AND A. B. PEVNYI

Abstract. We find the eigenvalues of modified Fourier matrix S with entries $S_{kj} = \frac{1}{\sqrt{n}}\omega^{k(1-j)}$, $0 \leqslant k, j \leqslant n-1$, where $\omega = \exp\frac{2\pi i}{n}$. For this matrix $S^4 = \omega I$. The matrix has an interesting property: for $n = 4m$ eigenvalues have equal multiplicities. We prove a theorem giving the multiplicities of eigenvalues for all n. The theorem is similar to Schur's theorem (1921) for standard Fourier matrix. Our proofs are self-contained. In the proof we calculate modified Gauss sums by means of the classical analysis.

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