

ON BERNSTEIN–TYPE INEQUALITIES FOR POLYNOMIALS INVOLVING THE POLAR DERIVATIVE

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Abstract. In this paper, we establish some upper bound estimates for the polar derivative of a polynomial not vanishing in a disk $|z| < k$, $k \geq 1$ with a zero of multiplicity s , $0 \leq s \leq n - 1$ at the origin. The obtained results enable us to derive polar derivative analogues of some well known Bernstein-type inequalities as special cases.

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