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A MOMENT PROBLEM IN A WEIGHTED L^2 SPACE ON THE REAL LINE

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Abstract. For a class of sets with multiple terms

$$\{\lambda_n,\mu_n\}_{n=1}^{\infty} := \{\underbrace{\lambda_1,\lambda_1,\dots,\lambda_1}_{\mu_1-times},\underbrace{\lambda_2,\lambda_2,\dots,\lambda_2}_{\mu_2-times},\dots,\underbrace{\lambda_k,\lambda_k,\dots,\lambda_k}_{\mu_k-times},\dots\},$$

we consider a moment problem of the form

$$\int_{-\infty}^{\infty} e^{-2w(t)}t^k e^{\overline{\lambda_n}t}f(t)\,dt = d_{n,k}, \quad \forall\, n\in\mathbb{N} \quad \text{and} \quad k=0,1,2,\dots\,\mu_n-1,$$

in a weighted $L^2(-\infty,\infty)$ space. We obtain a solution f which extends analytically as an entire function admitting a Taylor-Dirichlet series representation

$$f(z)=\sum_{n=1}^{\infty}\left(\sum_{k=0}^{\mu_n-1}c_{n,k}z^k\right)e^{\lambda_n z}, \quad c_{n,k}\in\mathbb{C}, \quad \forall\, z\in\mathbb{C}.$$

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